Shift Work and Business Cycles

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Abstract

Standard production function estimates suggest that productivity is procyclical. Significant empirical literature suggests that more accurately measuring utilization, of capital in particular, eliminates increasing returns to scale, and significantly reduces the appearance of procyclical productivity. Some suggest that changes in capital utilization are well-explained by changes in the number of shifts. This paper examines this claim by simulating a dynamic stochastic general equilibrium model that explicitly accounts for variable shift work. Results from simulations suggest that the inclusion of variable shift work does produce apparently procyclical productivity shocks even when shocks are to preferences or markups. However, these apparent productivity shocks are small relative to those that are observed in the data, and some business cycle moments are qualitatively different from those in the data unless strong assumptions are made about the cost of running an additional shift and the structure of the shocks. In contrast, the model that assumes that the number of shifts is fixed does qualitatively match those business cycle moments - when the model experiences true productivity shocks.

1 Introduction

A persistent puzzle in macroeconomics is the appearance of procyclical labor productivity. In attempting to estimate production functions, the coefficient on capital consistently shows the wrong sign. When capital is left out, the coefficient on labor tends to indicate increasing returns to scale - so during high production periods, labor is more productive than during low production periods. The real business cycle literature takes this fact as evidence of productivity shocks driving the business cycle (eg. Kydland and Prescott (1991)). However, some doubt that productivity shocks are a plausible explanation of business cycles, and offer a competing explanation: standard Solow residuals mismeasure total factor productivity because standard measures for labor and especially capital do not accurately represent these inputs.

A significant empirical literature explores this possibility, generally by attempting to replace the relatively poor measures of capital with measures that are considered to be more reflective of capital services. Burnside, Eichenbaum, and Rebelo (1995) argues that if one replaces a standard measure of capital (like those from the BEA's Fixed Asset tables) with electricity usage as a proxy for capital services, then one finds that Solow residuals are small relative to output variation and are not very strongly procyclical.

Susanto Basu (1996) reaches a similar conclusion. Basu observes that it is likely that output will be in fixed proportions with materials input (as Basu notes, a nail or sheet of steel cannot "work harder" or "run more hours" to increase output). Using this observation, Basu's estimates suggest that returns to scale are constant (though some of his estimates seem to suggest decreasing returns to scale). Also, he finds that cyclical utilization of labor and capital is significant - for each one percentage point increase in measured capital and labor, there is a 0.17 percent increase in utilization. Using his estimates, he finds that "true" technology shocks have a variance about 60 percent less than typical Solow residuals, and that the correlation between output and true technology shocks is about 75 percent lower than that for typical Solow residuals.

These two papers are typical of the empirical work done in this area, and certainly reach the typical conclusions. However, they leave the reader with a question: how does utilization vary on an operational level? Here, the work of Matthew Shapiro (1993, 1996b) is relevant. Shapiro notes that if the number of shifts operating is a variable that is a matter of choice (rather than being fixed or determined exogenously), then it is possible that the number of shifts in operation will vary systematically over the business cycle. This can easily explain where an increase in the utilization of capital may arise - after all, a single machine that runs 16 hours a day (over the course of two shifts) is certainly more highly utilized than a machine that operates for only 8 hours a day (over the course of a single shift). Taking this insight in hand, Shapiro (1993) finds that when the quantity of capital is adjusted for the number of hours of operation, procyclical productivity vanishes. In support of the idea that procyclical shift work may matter, Mayshar and Solon

(1993) find that shift work is strongly procyclical. In particular, the number of night shift workers is more closely related to the business cycle than the number of workers employed more generally. When combined with Shapiro's work, this suggests that procyclical shift work is an empirically plausible explanation for procyclical capital utilization.

This paper examines the implications of shift work in a simulated dynamic stochastic general equilibrium model. The intuition is simple: if changes in the incidence of shift work are the correct explanation for why productivity appears to be procyclical, then a simple model that explicitly incorporates shift work should be able to reproduce, at least qualitatively, important business cycle moments - in particular, the cyclicality of variables such as wages, investment, and employment, and the model should be able to reproduce these moments without requiring any true productivity shocks.

To a large degree, the model in this paper is a simpler, but somewhat less general form of the model presented in Mayshar and Halevy (1997). In their model, they allow for increasing returns to labor for small quantities. As a result, they have a U-shaped average labor cost curve. In the functional forms assumed in my model, returns to labor are always diminishing, so average labor costs are always increasing (I achieve a U-shaped average total cost curve in the second shift by assuming a fixed cost of running a second shift). One additional difference between Mayshar and Halevy (1997) and this paper is that this paper discusses the reaction of the capital stock as shocks work themselves out. This was not a point of interest for Mayshar and Halevy (1997), though it will be a very significant point of interest in this paper.

To my knowledge, the model that is most similar to the one presented here is Hornstein (2002). However, there are two signficant differences between the two papers. First, Hornstein (2002) emphasizes studying the difference between the workweek of capital and the workweek of labor. As such, he allows for shifts to be of variable length. This paper is less interested in the comparison between the workweek of capital and the workweek of labor, and as such I assume that shifts are of constant length - so a worker working at night works 40 hours a week, as does a worker working during the day. Secondly, in Hornstein (2002), the model is subjected to two shocks simultaneously, a government spending shock and a technology shock. The purpose was to show that the model performed relatively well when subjected to these two shocks simultaneously. The purpose of this paper is to examine the response of the economy when subjected to different kinds of shocks and to compare the responses. As such, the shocks will be applied separately.

In examining whether the shift work explanation works, this paper presents two basic models. In one, shift work is assumed to be constant. If the number of shifts is not an optimally chosen variable, then one can effectively act as if only a single shift is in operation, as changes in the number of shifts will obviously not affect the dynamics of the economy, since the number of shifts doesn't change. This one shift model is then shocked by four shocks: a preference shock regarding the leisure-consumption tradeoff, a productivity shock, a markup shock, and a shock to the discount factor. The dynamics that are most consistent with observed cyclicalities is a productivity shock. The second model is a model in which firms can pay a fixed cost to operate a second shift. The paper finds that, in absence of true productivity shocks, getting business cycle moments "right" is far from easy, and, in fact, the only variations of the model which fits with observed cyclicalities is one in which the labor-consumption preference shock is the primary driver, the preference shock is moderately persistent, and the fixed cost of operating a second shift is in terms of labor hours. However, in these cases, the volatility of the illusory "productivity shocks" is far below that observed in the data, even after doing robustness tests using a range of parameter values. In the end, if we accept the basic setup of this model and the commonly accepted cyclicalities of relevant variables, only the model in which productivity shocks play an important role can successfully reproduce the correct signs on relevant business cycle moments and get naively measured productivity shocks that are of a magnitude similar to those observed in the data.

The paper is organized as follows: Section 2 examines the empirical literature. Section 3 establishes some stylized facts related to the issue at hand. Section 4 presents the model. Section 5 presents the simulation technique and discusses the results. Section 6 presents conclusions and suggestions for possible future research.

2 The Empirical Literature

In Shapiro (1993), Matthew Shapiro estimates the relationship between conventional Solow residuals (ϵ) and share-weighted inputs (composed of labor hours, physical capital, energy, and materials, summed up as x). Using instrumental variables he obtains the following estimate:

$\Delta\epsilon = 0.57 + 0.31\Delta x + \Delta\epsilon*$

The coefficient on Δx is significantly positive at a one percent level - suggesting that there is a correlation between inputs and productivity. This is the standard result: total factor productivity is procyclical. Shapiro then suggests that, once capital is measured in terms of capital hours rather than in terms of the capital stock, this relationship weakens considerably - to the point where it is no longer statistically significant. To make this adjustment, Shapiro recalculates the Solow residual and the share-weighted inputs to include capital hours. Making these adjustments and using instrumental variables he obtains:

$\Delta\tilde{\epsilon} = 0.45 + 0.05\Delta\tilde{x} + \Delta\epsilon*$

The coefficient on Δx is not significant at any reasonable level. Therefore, it appears that measuring capital according to capital hours rather than physical capital eliminates procyclical total factor productivity. To further support his point, Shapiro estimates one last equation. He regresses conventional Solow residuals on conventional share-weighted inputs and the share of capital hours, accounting for them separately. Using this specification he obtains:

$\Delta\epsilon = 0.42 - 0.03\Delta x + 1.35\alpha_K \Delta s + \Delta\epsilon*$

So, once one includes capital hours in the regression, even conventionally measured Solow residuals are no longer correlated with conventionally measured inputs. Shapiro (1996b) finds similar results, but uses a number of different proxies for capital utilization. In the end, he concludes that capital hours perform the best - that is, they do the best job explaining away the relationship between Solow residuals and inputs.

Similarly, Burnside, Eichenbaum, and Rebelo (1995) find that using electricity consumption as a proxy for capital services works toward eliminating procyclical total factor productivity. In a simple exercise, they calculate Solow residuals using a labor share of 0.64 and a capital share of 0.36, and then consider two ways of measuring capital input: by the capital stock or by electricity consumption. When they use the capital stock, they find that Solow residuals are highly correlated with changes in output (correlation coefficient = 0.87). When they use electricity consumption, the correlation vanishes (correlation coefficient = 0.06). When they estimate the production function (using instrumental variables), they find a correlation between Solow residuals and changes in output of 0.31, which is far below that found using standard Solow residuals.

Basu and Kimball (1996) find similar results. In their paper, they use a model of firm level cost-minimization to eliminate any unobserved variables from their regression equation, and then use their estimates to obtain a measure of true technology shocks. They find that unobserved variations in input (for example, changes in labor and capital utilization) account for between 40 and 60 percent of the observed variations in total factor productivity.

In the end, the empirical literature suggests that procyclical productivity shocks are largely an illusion created by the mismeasurement of inputs - as correcting for this mismeasurement successfully eliminates much or all of the procyclical productivity.

3 Some Stylized Facts

Shiftwork has proven to be difficult to study explicitly, especially in a business cycle macroeconomic context, as no country keeps consistent records of shiftwork at quarterly frequencies. Even so, the literature has documented a number of stylized facts regarding shiftwork that a model of shiftwork should at the very least acknowledge.

3.1 Business Cycle Moments

The model will be tested by its ability to come near matching the relative standard deviations of GDP, labor input, wages, estimated productivity shocks (estimated by a naive Solow residual), and investment and the correlations of each of these variables with output. To obtain these moments, I used the NIPA tables to obtain real GDP and investment data, and the BLS series on employment and real weekly earnings. I used the Tangible Assets of Nonfarm Business from the Federal Reserve's Flow of Funds balance sheet tables as a measure of the capital stock[1], and the series on employment for a measure of labor input. Monthly data is averaged to obtain quarterly values, and all data series are then logged and HP-filtered using a smoothing coefficient of 1600 (which is standard for quarterly data). I calculate moments using the data from the first quarter of 1964 through the fourth quarter of 2008. To get an estimate of the Solow residual, I assume constant returns to scale and a capital share of .33, and then calculate the productivity shocks as $\hat{z} = \hat{Y} - .33\hat{K} - .67\hat{L}$, where Y, K, and L, are GDP, the capital stock, and employment, and hats indicate logged, detrended variables.

The moments are included as a column in Table 2. In general terms: labor, wages, investment, and estimated productivity shocks are all procyclical. Investment is more volatile than GDP. Labor input and wages are slightly less volatile than GDP, and estimated Solow residuals are approximately as volatile as GDP.

3.2 The Importance of Shiftwork in Capital Utilization

Using a combination of forms found in Hornstein (2002) and Bresnahan and Ramey (1992), one can think of quarterly production as occuring along these lines:

$$Production = \frac{Weeks}{Quarter}\frac{Days}{Week}\frac{Shifts}{Day}\frac{Hours}{Shift}\frac{Production}{Hour}$$

Here, the last term can be thought of as line speed, while the others are all changes in the hours of operation. The shift work explanation can be thought of in terms of $\frac{Shifts}{Day}$ being the most significant variable in capturing the varation in production. In an examination of automotive plants, Bresnahan and Ramey (1992) find that about 19.5 percent of output variation is the result of changes in the number of shifts. Only 6.3 percent is a result of changes in line speed. (The other 74.2 percent are mostly the result of changing the number of regular hours of operation - for example, planned shut downs for inventory adjustment - with a small contribution from changes in overtime hours.) Ramey and Vine (2007) find similar results. During three contractions in automotive output, approximately 25 percent of the variation in output was explained by changes in shifts - while line speed accounted for only 12.5 percent. Mattey and Strongin (1997) assume

[1]Replacing this series with the BEA's Fixed Asset tables, adjusted using investment data, does not significantly change the results.

that line speed is not an important margin of output variation and ignore it, focusing purely on capital hours. With that focus, they find that variations in the number of shifts explains approximately 73 percent of the contribution of capital hours to variations in production (and about 27 percent of the variance of production). These results suggest that shift work is the primary factor influencing capital utilization. In fact, in the observation of Carol Corrado in a response to Shapiro (1996), Corrado states: "[Shapiro] argues that the workweek of capital is a 'genuine measure of capital services' and uses the term interchangeably with 'capital utilization' and 'shift work.'" Therefore, it is reasonable to build a model that explicitly examines shift work to examine the hypothesis that shift work can produce illusory procyclical total factor productivity shocks similar to those seen in the data.

3.3 Wage Premium

The literature finds that there is, at most, a 25% wage premium for work done during a night shift. (This number was estimated by Shapiro (1996b).) Many studies find that there is no wage premium at all. However, these studies often suffer from a self-selection bias (that is, it is possible that low productivity workers choose to work at night to take advantage of a night wage premium at the individual level). After adjusting for self-selection, Kostiuk (1990) finds that the night wage premium is approximately 8%.

3.4 Workers Working at Night

Kostiuk (1990) reports that approximately 20% of workers work at night, while Mayshar and Halevy (1993) report that roughly 30% do.

3.5 Firms Operating Multiple Shifts

There is significant uncertainty regarding what share of firms operate multiple shifts. Shapiro(1996b) finds that the mean number of capital hours varies significantly depending on the data source. Data from the *Area Wage Survey* and *Current Population Survey* suggest that 37.5% of US firms operate multiple shifts, under certain assumptions[2], however the *Survey of Plant Capacity* implies a much larger incidence of shift work. Mattey and Strongin (1997) find that, according to the *Survey of Plant Capacity*, 73% of plants operate at least two shifts. Mayshar and Solon (1997) report that 70% of European industrial firms operate shifts, with night shifts being far smaller than day shifts. So, there is little consensus regarding how many firms operate multiple shifts.

[2]Shapiro (1996b) reports that the average number of capital hours per week in these surveys is roughly 55. If we assume the choice is either "one shift" or "two shifts", and that a single shift results in 40 capital hours a week, then 37.5 percent of firms employing two shifts would give the proper measure for capital hours. Allowing three shifts would decrease the inferred incidence of shiftwork. So, the 37.5 percent figure is an upper bound.

To get a starting grasp on some of the possible implications of these facts, it may be useful to consider a firm which faces the following short run decision, which is a simplified form of the decision described in the model below.

$$\max_{l_d, l_n}(l_d^{1-\alpha}k^\alpha - w_d l_d + l_n^{1-\alpha}k^\alpha - w_n l_n) \tag{1}$$

In this problem the firm has a simple Cobb-Douglas production function, a given level of capital, given wages, and must decide how much labor to employ during the day and during the night. Taking first order conditions and making substitutions, we can arrive at the following condition:

$$\frac{w_n}{w_d} = (\frac{l_d}{l_n})^\alpha \tag{2}$$

This equation can be used to arrive at some important facts.

1. If α=1/3 and the wage premium is 10%, then roughly 43% of workers work at night. If the wage premium is smaller, then more workers work at night.

2. If α=1/3 and 25% of workers work at night, then the wage premium must be 44%. A lower wage premium induces the hiring of more workers at night.

3. If the wage premium is 10% and the proportion of workers that work at night is 25%, then α=.09. A larger wage premium, or more workers working at night would tend to increase α.

So, in short, this very simple model has a difficult time reconciling the "moderate" levels of the stylized facts. However, it does come close to reconciling the wage premium and proportion of workers working at night if I assume the high wage premium of 25%, and the high proportion of night workers of 30%. However, a simple, plausible modification may be made by simply adding an extra parameter. Consider the following problem.

$$\max_{l_d, l_n}(l_d^{1-\alpha}k^\alpha - w_d l_d + (\gamma l_n)^{1-\alpha}k^\alpha - w_n l_n) \tag{3}$$

Here, we allow for nighttime labor to be more or less productive than daytime labor by introducing the multiplier γ. Now, we can fix the capital share, wage premium, and proportion of nighttime workers and allow the free parameter γ to bring the new form of the equation into balance.

$$\frac{w_n}{w_d} = \gamma^{1-\alpha}(\frac{l_d}{l_n})^\alpha \tag{4}$$

If we assume a 10% wage premium, 25% of workers working at night, and $\alpha = 1/3$, then $\gamma = .67$. That is, a single hour of nighttime work is roughly equivalent to .67 hours of daytime work. If we assume the higher values of a 25% wage premium, 30% of workers working at night, and $\alpha = 1/3$, then γ=.91. That is, a single hour of nighttime work is roughly equivalent to .91 hours of daytime work.

It is tempting to accept this simple modification to the model and leave the answer as "that is that". After all, this gap in productivity, though not insignificant, is certainly plausible[3]. However, it seems unlikely that an economy operating under these rules would show procyclical productivity. There is no incentive to change the number of shifts in operation, as the Inada condition that F'(0,k)=∞ guarantees that both shifts will operate. Since the number of shifts does not change, the channel that Shapiro suggests explains procyclical productivity will not be operative in this very simple model. To make this channel operative, the full model in this paper will introduce a fixed cost of operating a second shift.

4 Model

4.1 Model Overview

The model has 3 sets of agents: homogeneous households, homogeneous competitive final good producers, and a continuum of measure 1 of monopolistically competitive intermediate goods producers. Households provide daytime and nighttime labor which is used by the intermediate goods producers to produce their goods. The final good producers produce a homogeneous good which can be used for capital accumulation or consumption (the final good is also numeraire), using the intermediate goods as inputs (and acts as a "consumption aggregator" in this function). Intermediate goods producers have the most complex decision, as they must choose prices, the number of shifts operated, the quantity of labor hired in each operating shift, and the level of next period's capital stock.

4.2 Households

Households choose c, B', L_d, and L_n to solve the following problem:

$$h(B; K, \zeta) = \max_{c, L_d, L_n, B'} (U(c, L_d, L_n) + \beta\eta h(B'; K', \zeta'))$$

subject to:

$$c + \frac{B'}{R(K,\zeta)} \leq w_d(K,\zeta)L_d + w_n(K,\zeta)L_n + D(K,\zeta) + B$$

Where w_d is the wage for daytime labor, and w_n is the wage for nighttime labor. D is a dividend from firm profits. R is the gross interest rate paid on risk-free bonds, issued by households, that will return one unit of the numeraire (final) good next period. Each of these is a function of the aggregate state which is

[3]This is especially true if we accept the self-selection story that Kostiuk(1990) tells, and believe that the higher estimates for our stylized facts are more accurate.

composed of the aggregate capital stock K and a vector of shocks ζ. This problem will give rise to three Euler equations:

$-\frac{U_{L_d}}{U_c} = w_d$

$\frac{U_{L_d}}{U_{L_n}} = \frac{w_d}{w_n}$

$\frac{1}{R(K,\zeta)} = \beta\eta\frac{EU_c'}{U_c}$

The first governs the decision between consumption and (day) labor. The second governs the decision between night and day labor. The final Euler equation captures the intertemporal tradeoff.

4.3 Final Goods Firms

Final goods firms choose the quantity Y_j of each intermediate good purchased, taking prices as given. The final good is numeraire and can be used for consumption or investment in the capital stock. This takes the form of the following problem:

$\max\limits_{Y_j}(zF(Y) - \int\limits_0^1 P_j Y_j dj)$

Where Y is the set of all Y_j, and P_j is the price of intermediate good j, and z is an aggregate productivity shock. This problem gives the following first order condition, which is the demand curve for the intermediate good j.

$zF_{Y_j}(Y) = P_j$

4.4 Intermediate Goods Firms

The intermediate goods firms will have the following Bellman function:

$v(k_j, \phi_j; K, \zeta) = \max\limits_{P_j, k_j' \geq 0, l_{dj} \geq 0, l_{nj} \geq 0, s_j \in \{0,1\}} (P_j G(k_j, l_{dj}) - w_d l_{dj} - (k_j' - (1-\delta)k_j) + s_j(P_j G(k_j, l_{nj}) - w_n l_{nj} - \phi_j) + \frac{1}{R} EV(k_j', \phi_j'; K', \zeta'))$

subject to: $Y_j = G(k_j, l_{dj}) + s_j(G(k_j, l_{nj}))$

and $P_j = zF_{Y_j}(Y)$

Wages (w_n and w_d), aggregate output (Y), and the interest rate (R) are all functions of the aggregate state. ϕ_j is a stochastic fixed cost, in units of final goods, of operating a night shift. ϕ_j varies across firms at a point in time, and may vary across time for a single firm. Here, I assume that the same production function applies both during the day and during the night, for simplicity.

Solving this problem analytically is quite difficult because of the discrete choice of s_j, and the fact that that discrete choice will have impacts on P_j, and therefore all the other variables. To simplify the problem, we can rewrite the Bellman function, separating the current decision from the intertemporal one and substituting the equilibrium interest rate using the household problem. In this form, q denotes the marginal utility of consumption. For simplicity of notation, I suppress the aggregate state. Doing so, the problem becomes:

$V(k_j,\phi_j) = \max\{V_1(k_j,\phi_j), V_2(k_j,\phi_j)\} + \max_{k'_j \geq 0}(q(k'_j - (1-\delta)k_j)) + \beta\eta EV(k'_j,\phi'_j))$

$V_1(k_j,\phi_j) = \max_{P_j, l_{dj} \geq 0}(q(P_j G(k_j, l_{dj}) - w_d l_{dj}))$

$V_2(k_j,\phi_j) = \max_{P_j, l_{dj} \geq 0, l_{nj} \geq 0}(q(P_j G(k_j, l_{dj}) - w_d l_{dj} + (P_j G(k_j, l_{nj}) - w_n l_{nj} - \phi_j)))$

With the same constraints as before.

This setup gives some simple decision rules:

If $V_1 \geq V_2$, then $s_j = 0$ and $l_{nj} = 0$, and the other choice variables are chosen as solved in the V_1 problem.

If $V_2 > V_1$, then $s_j = 1$ and the other choice variables are chosen as solved in the V_2 problem.

We can note that the choice of k'_j is separated from the decision regarding labor and shifts. This effectively disentangles the present production problem from the investment problem, at the intermediate goods firm level.

From this problem, we can arrive at the following proposition: *There is a cutoff fixed cost such that, if the fixed cost experienced by a given firm is above that fixed cost, only a single shift will operate. If the fixed cost experienced by the given firm is below that fixed cost, two shifts will operate.* (Proof in the Appendix)

The following conditions describe the decisions for labor and capital by intermediate goods firms.

From V_1:

$P_j G_{l_{dj}}(k_j, l_{dj}) + \frac{\partial P_j}{\partial Y_j}\frac{\partial Y_j}{\partial l_{dj}} G(k_j, l_{dj}) = w_d$

From V_2:

$P_j G_{l_{dj}}(k_j, l_{dj}) + \frac{\partial P_j}{\partial Y_j}\frac{\partial Y_j}{\partial l_d j}(G(k_j, l_{dj}) + G(k_j, l_{nj})) = w_d$

$P_j G_{l_{nj}}(k_j, l_{nj}) + \frac{\partial P_j}{\partial Y_j}\frac{\partial Y_j}{\partial l_{nj}}(G(k_j, l_{dj}) + G(k_j, l_{nj})) = w_n$

From V:

$q = \beta\eta \frac{\partial}{\partial k'_j} EV(k'_j, \phi'_j)$

The first equates marginal revenue and marginal cost for day labor, in the case of a single shift, taking into account effects on prices. The second equates marginal revenue and marginal cost for day labor, in the case of two shift operation. The third equates marginal revenue and marginal cost for night labor. And the final equation equates the marginal value of current profits with the marginal value of additional capital in the next period.

4.5 Shocks

There are four sources of aggregate shocks in the model. First, there is the aggregate productivity shock. Second, I will introduce preference shocks regarding the leisure-consumption choice. Third, I will shock the degree of substitutability between intermediate goods. This shock acts as a markup shock. Fourth, the discount factor for future profits is a source of aggregate shocks.

4.6 Equilibrium

An equilibrium in this model is a set of functions $(R, w_n, w_d, L_d, L_n, c, B', P_j, Y_j, Y, l_{nj}, l_{dj}, s_j, k'_j, K)$ satisfying the following conditions:[4]

1. Firm and household decisions are optimal;
2. Markets for final goods, day and night labor, intermediate goods, and bonds clear;
3. The motion of aggregate capital is consistent with individual decisions regarding capital. That is, $K' = \int k'(K,\zeta)dj$.

5 Simulations

5.1 Functional Forms

To run simulations to obtain results, it is first necessary to specify forms for the various functions. For simulation purposes, I choose the following forms:

$U(c, L_d, L_n) = \log c - \gamma(\frac{\theta_d}{\theta_d+1} L_d^{(\theta_d+1)/\theta_d} + \psi \frac{\theta_n}{\theta_n+1} L_n^{(\theta_n+1)/\theta_n})$

γ captures the tradeoff between leisure and consumption. ψ captures the tradeoff between between day and night labor. θ_i captures the wage elasticity of labor supply for a particular shift. For purposes of simulations, γ will be considered as a possible source of aggregate shocks.

$F(Y) = (\int Y_j^{1/\lambda_f} dj)^{\lambda_f}$

F(Y) captures the physical input portion of final good production, and final good production is described by $zF(Y)$. This form is the same as that used by Christiano, Eichenbaum, and Evans (2005). λ_f will serve as a source of markup shocks.

$G(k, l) = k^\alpha l^{1-\alpha}$

This form is a standard Cobb-Douglas form with constant returns to scale, and capital share α.

Shocks will be assumed to follow an AR(1) process in logarithms. So that, for a shock process x, $\log x' = \rho_x \log x + \sigma_x \epsilon$

Finally, the distribution of ϕ is assumed to be uniform over an interval $[\underline{\phi}, \bar{\phi}]$. This form is chosen for computational simplicity, and is iid, so that every intermediate goods firm will have the same size of capital stock.

[4] To avoid excessive notation, I use choice variables to denote decision rules, and suppress the aggregate state.

5.2 Parameterization and Calibration

The following parameters must be set: $\alpha, \theta_d, \theta_n, \lambda_f, \bar{z}, \bar{\gamma}, \bar{\eta}, \psi, \beta, \underline{\phi}, \bar{\phi}, \rho_z, \rho_\gamma, \rho_\eta, \sigma_z, \sigma_\gamma, \sigma_\eta$.

α is set to the standard value of .33. θ_d and θ_n are set to unity, giving a labor supply elasticity of one in each shift. $\bar{z}$ and is normalized to unity. β is chosen to be .99, which would give a real annual interest rate of approximately 4 percent. $\underline{\phi}$ is set to zero to ensure that, in each period, some firms will operate two shifts.

The remaining parameters $(\lambda_f, \bar{\gamma}, \psi, \bar{\phi}, \rho_z, \rho_\gamma, \rho_\eta, \sigma_z, \sigma_\gamma, \sigma_\eta)$ will be chosen to match a number of relevant moments. In particular:

1. There should be a ten percent markup for the monopolistically competitive intermediate good producers. (This is a common moment.)

2. About 25 percent of workers should work at night.

3. Total work time should be roughly one third of the time endowment (which is normalized to one).

4. The night premium should be roughly ten percent.[5]

These four moments are used to pin down $\lambda_f, \bar{\gamma}, \psi$, and $\bar{\phi}$. For the shock parameters, I try a variety of values for ρ, and calibrate σ appropriately so that the standard deviation of output is roughly that which is seen in the data. Then, other moments can be calculated from the simulation.

5.3 Results

In order to get a handle on the illusory productivity shocks, it will be helpful to think of how productivity shocks are inferred, making standard assumptions. Assuming that output is a standard Cobb-Douglas function augmented by total factor productivity it is easy to derive the linearized equation:

$$\hat{z} = \hat{y} - \alpha\hat{k} - (1-\alpha)\hat{l} \tag{5}$$

With multiple shifts, the true equation is somewhat more complex. Here, I make the simplifying assumption that day shifts are the same size regardless whether a firm runs one or two shifts.[6]

$$Y = z(k^\alpha l_d^{1-\alpha} + \frac{\phi^T}{\bar{\phi}} k^\alpha l_n^{1-\alpha}) \tag{6}$$

When this equation is linearized, I obtain:

[5] This number is close to Kostiuk's estimate, and approximately midway between the estimate from Shapiro and that found in much of the literature.

[6] This assumption is false since the decision of how many shifts to run has an impact on the product price, but this fact has little impact on the logic that follows.

$$\hat{z} = \hat{y} - \alpha\hat{k} - \frac{\bar{l_d}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}(1-\alpha)\hat{l_d} - \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}(1-\alpha)\hat{l_n} - \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}\hat{\phi^T} \tag{7}$$

Consider what happens if we estimate z according to equation (5). In that case, we need a way to define $\hat{L}$ that is sensible. The following will serve this purpose well.

$$L = L_d + L_n \tag{8}$$

$$L = l_d + \frac{\phi^T}{\bar{\phi}} l_n \tag{9}$$

$$\hat{L} = \frac{\bar{l_d}}{\bar{l_d} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}}\hat{l_d} + \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}}{\bar{l_d} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}}(\hat{l_n} + \hat{\phi^T}) \tag{10}$$

Now we can calculate a productivity illusion that is the difference between an estimated $\hat{z^e}$ that is estimated under the assumption that there is a standard Cobb-Douglas function and the true $\hat{z}$. The productivity illusion can be described by the formula:

$$\hat{z^e} - \hat{z} = \kappa_1(1-\alpha)\hat{l_d} + \kappa_2(1-\alpha)\hat{l_n} + \kappa_3\hat{\phi^T} \tag{11}$$

$$\kappa_1 = \frac{\bar{l_d}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}} - \frac{\bar{l_d}}{\bar{l_d} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}} \tag{12}$$

$$\kappa_2 = \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}} - \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}}{\bar{l_d} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}} \tag{13}$$

$$\kappa_3 = \frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}}{\bar{l_d}^{1-\alpha} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}^{1-\alpha}} - (1-\alpha)\frac{\frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}}{\bar{l_d} + \frac{\bar{\phi^T}}{\bar{\phi}}\bar{l_n}} \tag{14}$$

It is easy to show that, as long as $\bar{l_n}$ is smaller than $\bar{l_d}$, κ_1 is negative. So, hiring more labor during the day will drive down the estimate of productivity. This is sensible. Since employment during the day is larger than at night, the marginal productivity of labor is relatively low. So, hiring more workers during the day drives down average productivity and the estimate for $\hat{z}$. For a similar reason, as long as $\bar{l_n}$ is smaller than $\bar{l_d}$, κ_2 is positive. Because fewer workers are hired at night, the marginal productivity of nighttime workers is relatively high. So, as the number of nighttime workers increases, average productivity increases. Finally, $\bar{l_n}$ being smaller than $\bar{l_d}$ is a sufficient condition[7] for establishing that κ_3 is positive. So, as the use of shiftwork increases, the estimates of productivity shocks will become larger.

[7]Actually, as long as l_n is not too far above l_d, this is still true.

Table 2 describes the results from a one-shift version of the model. The results are very clear along three lines.

First, a standard measure for the Solow Residual comes close to accurately identifying the shocks. When the shock is a preference shock (as in columns 1-3), the "estimated Solow residuals" are effectively zero. That is: the naive calculation for Solow residuals successfully shows that total factor productivity is actually constant in those models. This is also true when the shock is a markup shock (columns 7-9) or discount factor shock (columns 10-12). When the shock is a productivity shock (as in columns 4-6), the estimated Solow residuals have properties like the real shocks to the model[8]. Also, the only sets of cyclicality moments with signs that are consistent with the data are the sets generated by productivity shocks or some of the markup shocks.

If we assume that the economy operates on a single shift, and we see the appearance of procyclical productivity shocks, our conclusion is simple: there are procyclical productivity shocks.

Table 3 describes the results from the two-shift model where the costs are paid in final good units. (In the steady state, approximately 4 percent of output is spent on the fixed cost for operating a second shift.) There are several differences between the one shift and two shift model that are apparent by comparing Table 2 and Table 3.

First, in a one shift model, shocks to variables other than productivity do not lead to illusory productivity shocks. In the two shift model, they do - and, as long as the shocks are to the discount factor or leisure-consumption tradeoff and are not too persistent, the illusory productivity shocks are procyclical. This is true whether the shock is to the leisure-consumption tradeoff or whether it is to the discount factor. Sufficiently transitory shocks generate procyclical productivity. However, at the same time, transitory shocks generate countercyclical wages. Consider the case of a shock that makes leisure relatively less attractive. In that case, the supply of labor increases - pushing down wages. At the same time employment increases. So, wages will naturally be countercyclical in the first period. However, if the shock is sufficiently persistent, then the increase in labor supply will induce additional investment - which increases the productivity of labor and wages. So, with a sufficiently persistent shock, wages can be procyclical. However, in that case, the estimated productivity shock is countercyclical rather than procyclical. When the increase in wages occurs, fewer firms run multiple shifts. The result is that shift work is weakly procyclical (the initial increase is enough to offset the later decrease), but estimated productivity is countercyclical.

Next, consider a case where there is an increase in markups (that is, intermediate goods producers have more monopoly power). In this case, intermediate goods producers can increase their profit by decreasing

[8] The difference in standard deviations is because σz is the standard deviation of the shocks to the z process - which is AR(1), while the standard deviation of the estimated shocks does not take into account the AR(1) nature of the process.

output to drive up prices. As a result, output falls. However, at the same time, the higher markup makes it easier to cover the fixed cost of a second shift, so the proportion of firms operating a second shift increases. As a result, shift work is countercyclical, as are the estimated productivity shocks.

Finally, consider a case where there is an increase in the discount factor (that is, the economy becomes more patient). In this case, present consumption is not valued as highly and more resources are devoted to investment. The desire to keep consumption somewhat smooth while increasing investment leads to an increase in the supply of labor - which drives wages down. The fall in wages induces an increase in shift work and the illusion of a productivity shock. If the shock is transitory, then there is little continuing effect and wages are countercyclical (though weakly, as in the next period a higher capital stock drives up wages and output). If the shock is persistent, then the effect of capital accumulation is strong - and wages are procyclical and shiftwork (and measured productivity) are countercyclical. Thus, from either a shock to the leisure-consumption tradeoff or to the discount factor, it appears that the model cannot reconcile procyclical productivity with procyclical wages.

In order to give the best chance to the shift work explanation, I consider two variations to the model.

5.4 Variations

5.4.1 Labor Cost of a Shift

The first variation involves replacing the "final good" cost with a fixed "labor cost" for operating a second shift. So, rather than ϕ being a number thought of in terms of a quantity of the numeraire final good, it is a quantity of labor. The cost then is in terms of "$w_n\phi$". This model is then recalibrated, and the results are given in Table 4. For the most part, the results are very similar to those for a final good cost. However, there is one significant difference: as persistence of a preference shock increases, the countercyclicality of wages vanishes quickly while the procyclicality of the illusory productivity shocks does not vanish. The countercyclicality of wages vanishes quickly because a persistent shock leads to a large increase in the size of the capital stock, which, in turn increases the incidence of second shifts. An increase in shift work leads to the hiring not just of night time production workers, but also to the hiring of night time "overhead" workers. (In the steady state of this model, about 21 percent of night time labor is "overhead".) These two forces combine to make wages procyclical on the whole, rather than countercyclical, as long as the shock is sufficiently persistent. However, if the shock is too persistent, then the increase in the demand for productivity night labor will be enough to increase wages to the point where fewer firms will pay for overhead labor. So, in Column 3 of Table 4, wages are procyclical while shiftwork is very weakly countercyclical. However, with an appropriately chosen level of persistence (for example, $\rho_\gamma = 0.80$, as in Column 4), it is possible to balance these forces so that all of the cyclicalities have the proper sign.

With the markup and discount rate shocks, the story for this version of the model is similar to that in the final good cost version. Increased markups increase the incidence of shiftwork at the same time that output is declining. A transitory shock to the discount factor results in countercyclical wages, while a persistent shock to the discount factor results in countercyclical shift work.

Introducing a labor cost for a shift also has an impact on which level of persistence is reasonable for a productivity shock. In particular, if productivity shocks are too persistent, then the effect on wages is large enough that shift work turns countercyclical - even in the case where actual productivity has increased.

If we accept this version of the model, then we have two possibilities for the source of shocks that are qualitatively consistent with the cyclicality of business cycle moments: either shocks are somewhat persistent shocks to the labor-leisure tradeoff, or sufficiently transitory shocks to actual productivity. But, in the first case, estimated productivity shocks are about 5 percent the size seen in the data.

5.4.2 Capital Cost of a Shift

The second variation involves replacing the fixed final good cost of operating a shift with a capital cost of operating a shift. So, rather than paying ϕ to operate a second shift, the firm pays ϕk to operate a second shift. This sort of fixed cost can be thought of as additional wear and tear depreciation arising from using the physical capital goods more intensively. So, ϕ is a rate of additional depreciation that arises from operating a second shift, while δ captures rust and dust depreciation and wear and tear from operating a first shift. (In the steady state of the model, about 15 percent of depreciation is from this wear and tear during the second shift.) This variation is somewhat more intuitive than the basic variable shift model. (In the first model, it is not clear what the fixed cost actually is. All we know is that it is denominated in final good terms. In this version, it is clear that the cost arises from increased wear and tear on capital.)

A comparison between Table 5 and Table 3 reveals that the model with a final good cost and the model with a capital cost are very similar. In both, it is impossible to have a shock to the leisure-consumption tradeoff, markup, or discount rate that gives a set of moments that is consistent with known cyclicalities of major variables. In both, a productivity shock performs reasonably well in most cases. However, there is one significant difference: when the cost is in terms of capital, a highly persistent productivity shock results in countercyclical shift work.

Consider a shock to total factor productivity. Such a shock increases the marginal productivity of labor, which drives up wages, employment, and production. Because the increase in wages is smaller than the increase in productivity, there is an increase in the use of shift work. The productivity shock also increases the marginal productivity of capital, so that investment increases and capital accumulates. In this variation of the model, the increase in the capital stock causes a proportional increase in the fixed cost of shift work. So, if the effect from capital accumulation is sufficiently large (and that effect gets larger the more persistent the shocks are), then shift work will end up as countercyclical on the whole. In the model where the cost of an additional shift is in terms of increased depreciation of capital, we do see countercyclical shift work when shocks to productivity are very persistent.

In light of these results we are prepared to answer the question: can shift work provide a reasonable explanation for procyclical productivity, in absence of any actual shocks to productivity? If we restrict ourselves only to those versions of the model where the cyclicalities of major variables have the same sign as in the data, then we can say the following: (A) the cost of a second shift is overhead night labor, (B) shocks are moderately persistent shocks to the leisure-consumption choice. These two models do successfully reproduce the proper sign on the correlations that were studied in this paper. However, there is still reason to believe that these models do not provide a sufficient explanation for observed changes in productivity. In the data, the standard deviation of standard Solow residuals is approximately equal to the standard deviation of output. However, in these two versions of the model, the standard deviation of the estimated Solow residuals is no more than 5 percent the standard deviation of output.

In the end, one is left with two conclusions. First, variations in shift work can generate procyclical productivity, even when there are no true productivity shocks. Second, the variations of the Solow residuals generated by such a model are far smaller than those found in the data. Therefore, mismeasurement of capital because of shift work is an insufficient explanation for procyclical productivity.

5.5 Alternative Parameters

To strengthen the case that shift work will not create a large enough variance for Solow residuals, I now turn to considering a variety of ranges for my parameters. For this exercise, I considered the case where the cost of a second shift is in the form of a labor cost, where the shock is to γ, and is of a moderately high persistence ($\rho_\gamma = 0.80$). Then, I considered a range for each parameter, resimulated the model using various elements in that range, and found the maximum σ_{z^e}/σ_Y, discarding any simulations that produced a moment with the wrong sign. Throughout the exercise, I did not readjust other parameters so that my original calibration moments held. I adjusted along one margin at a time. The results are reported in Table 6.

Most of the changes made little difference, suggesting that the results are quite robust. However, setting a high level for α of 0.70 did give a relative standard deviation for the shocks that is similar to what is seen in the data. This level for α will probably be seen as immediately implausible, if only because it approximately reverses the capital and labor shares of income that are observed in macroeconomic data. In addition to this cause for concern, there are others. With this alternative parameterization, the 45 percent of output is used to maintain the capital stock, as the capital-output ratio is quite high. The steady state night wage premium is 96 percent, which is far above even the high estimates obtained by Shapiro (1996b). 55 percent of night labor is overhead. Also, while the relative standard deviation of the naive Solow residual compared to output is approximately correct, the relative standard deviation of labor input is far from the data. In the data, the standard deviation of labor is about 83 percent that of GDP. In this high alpha version of the model, that ratio falls to 5 percent. In short: this seems to solve the problem for the size of the illusory productivity shock, but creates a problem for the volatility of employment.

The alternative parameterization provides a useful clue to the fundamental problem that prevents the model from being able to match the data. If output is produced through a combination of total factor productivity, labor, and capital, for total factor productivity to have a standard deviation roughly equal to the standard deviation of output, labor and capital must not vary at all - or one must be countercyclical, so that the variations in capital and labor cancel one another out. When alpha rises to this level, the standard deviation of labor is 5 percent that of output, and the standard deviation of capital is about 10 percent that of output. So, even if these two were perfectly correlated, there is still substantial variation to be explained by changes in productivity.

6 Conclusions and Extentions

The central conclusion of this paper is two-fold. On the one hand, the paper does suggest that procyclical shift work is capable of producing the illusion of procyclical productivity shocks, when standard Solow residuals are used for these purposes. On the other hand, the paper suggests that procyclical shift work is unlikely to be a good explanation for procyclical productivity because the model which allows for procyclical shift work generally implies other business cycle moments that contradict the data, qualitatively. Even when business cycle moments show the correct sign, the standard deviation of estimated productivity shocks is much smaller than that observed the data. This suggests that the mismeasurement of capital because of cyclical variations in shift work is an insufficient explanation for procyclical productivity.

One possibility that the paper does not explore is that some plants may have variable shifts while others operate a fixed number of shifts. A priori, one cannot rule out that this variation may have different results than those found in this paper. However, it seems unlikely that such a change will significantly improve the results, as it would likely decrease the variance of shiftwork, which, in turn, would decrease the variance of the Solow residuals moving the model further from the data. Also, in the model as it is, there are some firms that will always operate one shift, and some that will always operate two shifts. Changing this fact from being the result of an optimal choice to being imposed by the model itself seems unlikely to cause significant changes.

A second possible extension would be to consider other shocks. In this paper, I assumed that shocks were in some “real” form - preferences, markups, or productivity. One could take the central feature of this model (a fixed cost for night shift work) and apply it in monetary models. Such a model is likely to be more complicated than the model offered here, if money is to play a meaningful role. However, one can get a rough idea of what we may expect to happen using the insights from this model. For money to play a significant role, some price and wage stickiness is required. If price stickiness is less severe than wage stickiness, then a monetary shock would increase markups. The effect of the increased markup is likely to look very much like a shock to my λ_f variable. However, there will be one significant difference: real wages will fall when output increases. So, real wages will be countercyclical, rather than procyclical (as in the model here). At the same time, interest rates will fall. This fall is very similar to an increase in η - which tends to increase output and wages if it is sufficiently persistent. So, the question is whether the combination will be able to increase real wages and, at the same time, lead to an increase in shift work that generates proyclical productivity. Even if the model does succeed in qualitatively matching business cycle moments, it seems unlikely that it will be able to generate procyclical Solow residuals near the magnitude that we see in the data.

In the end, this paper suggests that procyclical shift work is unlikely to provide a good explanation for the appearance of procyclical productivity. While procyclical shift work does actually create that illusion, it also results in some business cycle moments that are substantially different from those found in the data - either some important correlation between output and another variable has the wrong sign, or the variance of Solow residuals is too low. Future work in this area should consider alternative explanations for procyclical productivity, as shiftwork-related mismeasurement of factors is an insufficient explanation.

7 Tables

Parameter	One-Shift	Final Good Cost	Labor Cost	Capital Cost
α	0.33	0.33	0.33	0.33
$\bar{z}$	1	1	1	1
$\bar{\eta}$	1	1	1	1
β	0.99	0.99	0.99	0.99
δ	0.025	0.025	0.025	0.025
$\underline{\phi}$	NA	0	0	0
λ_f	1.1	1.1	1.1	1.1
θ_d	1	1	1	1
θ_n	1	1	1	1
$\bar{\gamma}$	6.9714	9.3618	9.3618	9.3618
ψ	3.3	3.3	3.3	3.3
$\bar{\phi}$	NA	0.4378	0.2350	0.0380

Table 1: Parameters

Variable	Data	1	2	3	4	5	6
σ_γ	-	0.0305	0.0256	0.0112	-	-	-
ρ_γ	-	0	0.5	0.9	-	-	-
σ_z	-	-	-	-	0.0102	0.0086	0.0039
ρ_z	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0147	0.0145	0.0141	0.0149	0.0147	0.0137
St. Dev. Labor	0.0125	0.0219	0.0210	0.0169	0.0070	0.0064	0.0039
St. Dev. Wage	0.0118	0.0075	0.0072	0.0053	0.0081	0.0089	0.0109
St. Dev. ProdShockEst	0.0155	0.0000	0.0000	0.0000	0.0102	0.0099	0.0088
St. Dev. Inv	0.0689	0.0655	0.0610	0.0405	0.0663	0.0613	0.0407
Corr(GDP, Labor)	0.8063	0.9928	0.9837	0.9561	0.9811	0.9447	0.7956
Corr(GDP, Wage)	0.4472	-0.9369	-0.8505	-0.3801	0.9859	0.9719	0.9758
Corr(GDP, ProdShockEst)	0.9017	-0.5443	0.8889	0.3844	0.9968	0.9916	0.9824
Corr(GDP, Inv)	0.5698	0.9877	0.9712	0.8974	0.9886	0.9675	0.9028
Variable	Data	7	8	9	10	11	12
σ_{λ_f}	-	0.0166	0.0139	0.0060	-	-	-
ρ_{λ_f}	-	0	0.5	0.9	-	-	-
σ_η	-	-	-	-	0.0314	0.0131	0.0014
ρ_η	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0148	0.0151	0.0162	0.0152	0.0147	0.0149
St. Dev. Labor	0.0125	0.0220	0.0216	0.0184	0.0221	0.0193	0.0123
St. Dev. Wage	0.0118	0.0231	0.0234	0.0249	0.0092	0.0095	0.0109
St. Dev. ProdShockEst	0.0155	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
St. Dev. Inv	0.0689	0.0658	0.0647	0.0543	0.1772	0.1550	0.1015
Corr(GDP, Labor)	0.8063	0.9925	0.9810	0.9365	0.9450	0.8771	0.6924
Corr(GDP, Wage)	0.4472	1.000	0.9999	0.9998	-0.6194	-0.2270	0.5833
Corr(GDP, ProdShockEst)	0.9017	-0.4786	0.4811	0.2389	0.3982	-0.4652	0.2200
Corr(GDP, Inv)	0.5698	0.9873	0.9673	0.8802	0.9538	0.8982	0.7650

Table 2: Results from One Shift Model

Variable	Data	1	2	3	4	5	6
σ_γ	-	0.0273	0.0232	0.0106	-	-	-
ρ_γ	-	0	0.5	0.9	-	-	-
σ_z	-	-	-	-	0.0091	0.0077	0.0047
ρ_z	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0150	0.0149	0.0151	0.0152	0.0161	0.0204
St. Dev. Labor	0.0125	0.0209	0.0201	0.0184	0.0074	0.0076	0.0087
St. Dev. Wage	0.0118	0.0069	0.0078	0.0095	0.0095	0.0122	0.0220
St. Dev. PhiT	-	0.0085	0.0082	0.0072	0.0086	0.0089	0.0096
St. Dev. ProdShockEst	0.0155	0.0011	0.0013	0.0015	0.0102	0.0104	0.0110
St. Dev. Inv	0.0689	0.0629	0.0574	0.0405	0.0639	0.0624	0.0536
Corr(GDP, Labor)	0.8063	0.9965	0.9916	0.9856	0.9999	0.9996	0.9967
Corr(GDP, Wage)	0.4472	-0.6899	-0.3195	0.3461	0.9435	0.9142	0.9471
Corr(GDP, PhiT)	0.9017	0.8855	0.6971	0.1666	0.8890	0.7071	0.1133
Corr(GDP, ProdShockEst)	0.9017	0.6959	0.3285	-0.3386	0.9801	0.9489	0.8821
Corr(GDP, Inv)	0.5698	0.9848	0.9608	0.9075	0.9852	0.9620	0.9039

Variable	Data	7	8	9	10	11	12
σ_{λ_f}	-	0.0492	0.0391	0.0135	-	-	-
ρ_{λ_f}	-	0	0.5	0.9	-	-	-
σ_η	-	-	-	-	0.0281	0.0110	0.0011
ρ_η	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0150	0.0154	0.0148	0.0151	0.0153	0.0141
St. Dev. Labor	0.0125	0.0246	0.0233	0.0180	0.0205	0.0183	0.00135
St. Dev. Wage	0.0118	0.0263	0.0279	0.0284	0.0111	0.0154	0.0181
St. Dev. PhiT	-	0.1121	0.1051	0.0774	0.0103	0.0115	0.0118
St. Dev. ProdShockEst	0.0155	0.0022	0.0028	0.0034	0.0018	0.0025	0.0029
St. Dev. Inv	0.0689	0.1071	0.0979	0.0641	0.1545	0.1295	0.0768
Corr(GDP, Labor)	0.8063	0.9877	0.9712	0.9590	0.9825	0.9611	0.9495
Corr(GDP, Wage)	0.4472	0.9988	0.9977	0.9982	-0.2046	0.3549	0.8005
Corr(GDP, PhiT)	0.9017	-0.9825	-0.9577	-0.9334	0.5437	-0.0449	-0.6790
Corr(GDP, ProdShockEst)	0.9017	-0.9459	-0.9285	-0.9618	0.2115	-0.3503	-0.7990
Corr(GDP, Inv)	0.5698	0.9636	0.9059	0.8048	0.9419	0.8479	0.7374

Table 3: Results from Basic Model - Final Good Cost

Variable	Data	1	2	3	4	5	6	7
σ_γ	-	0.0262	0.0221	0.0102	0.0150	-	-	-
ρ_γ	-	0	0.5	0.9	0.8	-	-	-
σ_z	-	-	-	-	-	0.0097	0.0088	0.0047
ρ_z	-	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0145	0.0144	0.0148	0.0153	0.0146	0.0159	0.0186
St. Dev. Labor	0.0125	0.0204	0.0192	0.0165	0.0185	0.0069	0.0070	0.0058
St. Dev. Wage	0.0118	0.0061	0.0066	0.0077	0.0078	0.0087	0.0110	0.0177
St. Dev. PhiT	-	0.0106	0.0104	0.0095	0.0105	0.0053	0.0073	0.0121
St. Dev. ProdShockEst	0.0155	0.0008	0.0007	0.0006	0.0007	0.0099	0.0104	0.0108
St. Dev. Inv	0.0689	0.0640	0.0584	0.0399	0.0510	0.0644	0.0644	0.0505
Corr(GDP, Labor)	0.8063	0.9917	0.9799	0.9594	0.9632	0.9932	0.9841	0.9735
Corr(GDP, Wage)	0.4472	-0.7491	-0.4250	0.3542	0.0510	0.9611	0.9399	0.9626
Corr(GDP, PhiT)	-	0.8651	0.6561	-0.0382	0.2662	0.5056	0.0904	-0.5913
Corr(GDP, ProdShockEst)	0.9017	0.9876	0.9693	0.9324	0.9415	0.9899	0.9759	0.9519
Corr(GDP, Inv)	0.5698	0.9834	0.9578	0.8886	0.9131	0.9824	0.9562	0.8890
Variable	Data	8	9	10	11	12	13	
σ_{λ_f}	-	0.0512	0.0461	0.0420	-	-	-	
ρ_{λ_f}	-	0	0.5	0.9	-	-	-	
σ_η	-	-	-	-	0.0274	0.0109	0.0010	
ρ_η	-	-	-	-	0	0.5	0.9	
St. Dev. GDP	0.0150	0.0150	0.0150	0.0149	0.0149	0.0150	0.0150	
St. Dev. Labor	0.0125	0.0228	0.0220	0.0255	0.0200	0.0170	0.0108	
St. Dev. Wage	0.0118	0.0226	0.0250	0.0451	0.0098	0.0126	0.0159	
St. Dev. PhiT	-	0.0987	0.1040	0.1915	0.0139	0.0158	0.0181	
St. Dev. ProdShockEst	0.0155	0.0004	0.0005	0.0015	0.0008	0.0006	0.0004	
St. Dev. Inv	0.0689	0.0661	0.0549	0.0121	0.1587	0.1329	0.0765	
Corr(GDP, Labor)	0.8063	0.9926	0.9820	0.9961	0.9510	0.8927	0.8554	
Corr(GDP, Wage)	0.4472	0.9998	1.000	0.9992	-0.2045	0.3209	0.8348	
Corr(GDP, PhiT)	-	-0.9960	-0.9899	-0.9970	0.4230	-0.1289	-0.7761	
Corr(GDP, ProdShockEst)	0.9017	-0.9959	-0.9887	-0.9965	0.9259	0.8279	0.6932	
Corr(GDP, Inv)	0.5698	0.9852	0.9602	0.9999	0.9287	0.8352	0.7140	

Table 4: Results from Model with Labor Cost

Variable	Data	1	2	3	4	5	6
σ_γ	-	0.0270	0.0227	0.0103	-	-	-
ρ_γ	-	0	0.5	0.9	-	-	-
σ_z	-	-	-	-	0.0090	0.0079	0.0043
ρ_z	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0150	0.0152	0.0150	0.0151	0.0157	0.0183
St. Dev. Labor	0.0125	0.0208	0.0206	0.0186	0.0074	0.0077	0.0087
St. Dev. Wage	0.0118	0.0065	0.0074	0.0096	0.0090	0.0109	0.0184
St. Dev. PhiT	-	0.0103	0.0123	0.0169	0.0104	0.0127	0.0205
St. Dev. ProdShockEst	0.0155	0.0015	0.0019	0.0027	0.0101	0.0101	0.0098
St. Dev. Inv	0.0689	0.0503	0.0483	0.0364	0.0505	0.0498	0.0442
Corr(GDP, Labor)	0.8063	0.9979	0.9949	0.9881	0.9994	0.9983	0.9937
Corr(GDP, Wage)	0.4472	-0.7461	-0.4409	0.2891	0.9509	0.9182	0.9309
Corr(GDP, PhiT)	-	0.6936	0.3617	-0.3518	0.6844	0.3553	-0.3609
Corr(GDP, ProdShockEst)	0.9017	0.6164	0.2610	-0.4180	0.9743	0.9349	0.7923
Corr(GDP, Inv)	0.5698	0.9904	0.9756	0.9275	0.9901	0.9753	0.9275
Variable	Data	7	8	9	10	11	12
σ_{λ_f}	- 0.0590	0.0496	0.0295	-	-	-	
ρ_{λ_f}	- 0	0.5	0.9	-	-	-	
σ_η	-	-	-	-	0.0268	0.0109	0.0009
ρ_η	-	-	-	-	0	0.5	0.9
St. Dev. GDP	0.0150	0.0148	0.0154	0.0147	0.0151	0.0150	0.0152
St. Dev. Labor	0.0125	0.0295	0.0285	0.0265	0.0202	0.0188	0.0151
St. Dev. Wage	0.0118	0.0300	0.0330	0.0443	0.0105	0.0131	0.0191
St. Dev. PhiT	-	0.1333	0.1358	0.1669	0.0178	0.0227	0.0336
St. Dev. ProdShockEst	0.0155	0.0053	0.0064	0.0089	0.0028	0.0036	0.0054
St. Dev. Inv	0.0689	0.0480	0.0402	0.0038	0.1125	0.1006	0.0620
Corr(GDP, Labor)	0.8063	0.9909	0.9709	0.9523	0.9875	0.9772	0.9722
Corr(GDP, Wage)	0.4472	1.000	0.9999	0.9881	-0.1914	0.1886	0.7996
Corr(GDP, PhiT)	-	-0.9930	-0.9764	-0.9475	0.1275	-0.2377	-0.8121
Corr(GDP, ProdShockEst)	0.9017	-0.9921	-0.9879	-0.9998	0.0525	-0.2915	0.8251
Corr(GDP, Inv)	0.5698	0.9735	0.9055	-0.1493	0.9490	0.8973	0.7712

Table 5: Results from Capital Cost

Parameter	Min	Max	x*	max σ_{z^e}/σ_Y
α	0.10	0.90	0.70	1.0137
β	0.59	0.99	0.94	0.0614
δ	0.01	0.10	0.02	0.0512
ψ	1.00	5.00	1.00	0.1109
λ_f	1.01	2.01	1.11	0.0440
$\bar{\phi}$	0.10	0.90	0.10	0.0626
$\bar{\gamma}$	5	15	5	0.0856
θ_n	0.50	2.00	0.70	0.0802
θ_d	0.50	2.00	0.50	0.0974

Table 6: Alternative Parameters

8 Appendix: Proof of Proposition

Proposition: *There is a cutoff fixed cost such that, if the fixed cost experienced by a given firm is above that fixed cost, only a single shift will operate. If the fixed cost experienced by the given firm is below that fixed cost, two shifts will operate.*

Formally, this proposition may be stated (Here I suppress k_j, as it is held constant.): If $\phi_1 > \phi_2$ and $V_2(\phi_1) > V_1(\phi_1)$, then $V_2(\phi_2) > V_1(\phi_2)$. If $\phi_1 > \phi_2$ and $V_1(\phi_2) > V_2(\phi_2)$, then $V_1(\phi_1) > V_2(\phi_1)$. This proposition follows directly from the following observations:

Observation 1: V_1 does not depend on ϕ. Thus, $V_1(\phi_1) = V_1(\phi_2)$.

Observation 2: The optimal choices for the choice variables when $\phi = \phi_1$ are also available when $\phi = \phi_2$. A non-optimizing entrepreneur experiencing ϕ_2 could mimic an entrepreneur optimizing under ϕ_1 and have a higher V_2. Sensibly, an optimizing entrepreneur will do no worse than this non-optimizer. So, $V_2(\phi_2) > V_2(\phi_1)$.

With these two observations and the assumptions from the proposition we have:

If $V_2(\phi_1) > V_1(\phi_1)$, $V_2(\phi_2) > V_2(\phi_1) > V_1(\phi_1) = V_1(\phi_2)$, which implies: $V_2(\phi_2) > V_1(\phi_2)$.

If $V_1(\phi_2) > V_2(\phi_2)$, $V_1(\phi_1) = V_1(\phi_2) > V_2(\phi_2) > V_2(\phi_1)$, which implies: $V_1(\phi_1) > V_2(\phi_1)$.

Assuming that ϕ is continuous, it follows that there is a cutoff $\bar{\phi}$, where $V_1(\bar{\phi}) = V_2(\bar{\phi})$. If $\phi > \bar{\phi}$, then $V_1(\phi) > V_2(\phi)$. If $\phi < \bar{\phi}$, then $V_2(\phi) > V_1(\phi)$. qed

Note: If ϕ is bounded, then it may be that $V_1 > V_2$ or $V_2 > V_1$ for all ϕ within those bounds.

9 Functional Forms

For household (simplified to account for the fact that, in equilibrium, bond issue is zero):

$U = lnC - \gamma(\frac{\theta_d}{\theta_d+1}L_d^{(\theta_d+1)/\theta_d} + \frac{\psi\theta_n}{\theta_n+1}L_n^{(\theta_n+1)/\theta_n})$

Subject to:

$C \leq w_d L_d + w_n L_n + D$

FOCs:

$\gamma L_d^{1/\theta_d} = \frac{w_d}{C}$

$\gamma\psi L_n^{1/\theta_n} = \frac{w_n}{C}$

For final good firms:

$\max\limits_{Y_j} z(\int\limits_0^1 Y_j^{1/\lambda_f} dj)^{\lambda_f} - \int\limits_0^1 P_j Y_j dj$

FOC:

$z(\frac{Y_j}{Y})^{1/\lambda_f - 1} = P_j$ Where $Y = \int\limits_0^1 Y_j^{1/\lambda_f} dj)^{\lambda_f}$

For intermediate good firms:

$V_1 = \max P_j l_{dj}^{1-\alpha} k_j^{\alpha} - w_d l_{dj}$

Where:

$P_j = z(\frac{Y_j}{Y})^{1/\lambda_f - 1}$

and

$Y_j = l_{dj}^{1-\alpha} k_j^{\alpha}$

FOC (much simplified):

$l_{dj} = (\frac{\lambda_f w_d Y^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)}$

$V_2 = \max P_j l_{dj}^{1-\alpha} k_j^{\alpha} - w_d l_{dj} + P_j l_{nj}^{1-\alpha} k_j^{\alpha} - w_n l_{nj} - \phi_j$

Where:

$P_j = z(\frac{Y_j}{Y_t})^{1/\lambda_f - 1}$

and

$Y_j = l_{dj}^{1-\alpha} k_j^{\alpha} + l_{nj}^{1-\alpha} k_j^{\alpha}$

FOCs (much simplified):

$l_{dj} = (\frac{w_d}{w_n})^{-1/\alpha} l_{nj}$

$l_{nj} = (\frac{\lambda_f w_n (Y/((\frac{w_d}{w_n})^{1-1/\alpha}+1))^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)}$

Result: there is a cutoff fixed cost ϕ^T such that if it's below that, the firm operates 2 shifts. Above, 1 shift.

That can be found using an equation of this form (substituting the above) as necessary and relevant.

$\phi^T = P_2 Y_{d2} - w_d l_{d2} + P_2 Y_{n2} - w_n l_{n2} - P_1 Y_{d1} + w_d l_{d1}$

We can also get Y_t simplified using this knowledge. So,

$Y = (\frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} Y_2^{1/\lambda_f} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} Y_1^{1/\lambda_f})^{\lambda_f}$

Then we need some resource constraints:

Final goods: $zY = C + 1/2(\phi^{T2} - \underline{\phi}^2) + K' - (1-\delta)K$

Intermediate goods: taken care of automatically, since notation does not change.

Labor: $L_d = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{d2} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} l_{d1}$

$L_n = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{n2}$

The entire system of equations:

$$\frac{w_d}{C} = \gamma L_d^{1/\theta_d} \tag{1}$$

$$\frac{w_n}{C} = \psi\gamma L_n^{1/\theta_n} \tag{2}$$

$$P_1 = z(\frac{Y_1}{Y})^{1/\lambda_f - 1} \tag{3}$$

$$P_2 = z(\frac{Y_2}{Y})^{1/\lambda_f - 1} \tag{4}$$

$$l_{d2} = (\frac{w_d}{w_n})^{-1/\alpha} l_{n2} \tag{5}$$

$$l_{n2} = (\frac{\lambda_f w_n (Y/((\frac{w_d}{w_n})^{1-1/\alpha} + 1))^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{6}$$

$$l_{d1} = (\frac{\lambda_f w_{dt} Y^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{7}$$

$$q = \beta\eta \frac{\partial}{\partial k'} E_\phi V(k', \phi') \tag{8}$$

$$\phi^T = P_2 Y_2 - w_d l_{d2} - w_n l_{n2} - P_1 Y_1 + w_d l_{d1} \tag{9}$$

$$L_d = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{d2} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} l_{d1} \tag{10}$$

$$L_n = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{n2} \tag{11}$$

$$Y = (\frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} Y_2^{1/\lambda_f} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} Y_1^{1/\lambda_f})^{\lambda_f} \tag{12}$$

$$zY = C + \frac{(\phi^{T2} - \underline{\phi}^2)}{2(\bar{\phi} - \underline{\phi})} + k' - (1-\delta) * k \tag{13}$$

$$Y_1 = l_{d1}^{1-\alpha} k^\alpha \tag{14}$$

$$Y_2 = l_{d2}^{1-\alpha} k^\alpha + l_{n2}^{1-\alpha} k^\alpha \tag{15}$$

$$q = \frac{1}{C} \tag{16}$$

Linearized:

$$\hat{w_d} - \hat{C} = \hat{\gamma} + \frac{1}{\theta_d}\hat{L_d} \tag{1'}$$

$$\hat{w_n} - \hat{C} = \hat{\psi} + \hat{\gamma} + \frac{1}{\theta_n}\hat{L_n} \tag{2'}$$

$$\hat{P}_1 = \hat{z} + \frac{1-\lambda_f}{\lambda_f}(\hat{Y}_1 - \hat{Y}) - \ln(\frac{Y_1}{Y})\frac{1}{\lambda_f^2}\hat{\lambda_f} \tag{3'}$$

$$\hat{P}_2 = \hat{z} + \frac{1-\lambda_f}{\lambda_f}(\hat{Y}_2 - \hat{Y}) - \ln(\frac{Y_2}{Y})\frac{1}{\lambda_f^2}\hat{\lambda_f} \tag{4'}$$

$$\hat{l_{d2}} = -\frac{1}{\alpha}(\hat{w_d} - \hat{w_n}) + \hat{l_{n2}} \tag{5'}$$

$$\begin{aligned}
\hat{l_{n2}} =& (\frac{\lambda_f}{1-\alpha-\lambda_f})(\hat{w_n} + \frac{1-\lambda_f}{\lambda_f}\hat{Y} - \hat{z} - \hat{k}) + (\frac{\lambda_f \bar{w_n}\bar{Y}^{1/\lambda_f - 1}}{(1-\alpha)\bar{z}\bar{k}^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)}(\frac{1-\lambda_f}{1-\alpha-\lambda_f})... \\
&...((\frac{\bar{w_d}}{\bar{w_n}})^{1-1/\alpha} + 1)^{-\lambda_f/(1-\alpha-\lambda_f)}(\frac{\bar{w_d}}{\bar{w_n}})^{1-1/\alpha}(1 - \frac{1}{\alpha})(\hat{w_d} - \hat{w_n})... \\
&... + \frac{\lambda_f}{(1-\alpha-\lambda_f)^2}(1-\alpha-\lambda_f + (1-\alpha)\ln(\lambda_f) + (1-\alpha)\ln(\frac{w_n}{(1-\alpha)z})... \\
&... + \alpha\ln(\frac{Y}{(\frac{w_d}{w_n})^{1-1/\alpha}+1}) + \alpha\ln(\frac{1}{K}))\hat{\lambda_f}
\end{aligned} \tag{6'}$$

$$\begin{aligned}
\hat{l_{d1}} =& \frac{\lambda_f}{1-\alpha-\lambda_f}(\hat{w_d} + \frac{1-\lambda_f}{\lambda_f}\hat{Y} - \hat{z_t} - \frac{\alpha}{\lambda_f}\hat{k}) + \frac{\lambda_f}{(1-\alpha-\lambda_f)^2}(1-\alpha-\lambda_f... \\
&... + (1-\alpha)\ln(\lambda_f) + (1-\alpha)\ln(\frac{w_d}{(1-\alpha)z}) + \alpha\ln(Y) + \alpha\ln(\frac{1}{K}))\hat{\lambda_f}
\end{aligned} \tag{7'}$$

$$\begin{aligned}
\hat{q} =& \hat{\eta} + E[\hat{q}' + \beta\frac{\bar{\phi} - \bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\frac{\alpha}{\bar{\lambda}_f}\frac{\bar{P}_1\bar{Y}_1}{\bar{k}}(\hat{P}_1' + \hat{Y}_1')... \\
&... + \beta\frac{\bar{\phi}^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\frac{\alpha}{\bar{\lambda}_f}\frac{\bar{P}_2\bar{Y}_2}{\bar{k}}(\hat{P}_2' + \hat{Y}_2') \\
&... + \beta\frac{\bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\frac{\alpha}{\bar{\lambda}_f}\frac{\bar{P}_2\bar{Y}_2 - \bar{P}_1\bar{Y}_1}{\bar{k}}\hat{\phi^T}'... \\
&... - \beta\frac{\alpha}{\bar{\lambda}_f}\frac{(\bar{\phi} - \bar{\phi}^T)\bar{P}_1\bar{Y}_1 + (\bar{\phi}^T - \underline{\phi})\bar{P}_2\bar{Y}_2}{(\bar{\phi} - \underline{\phi})\bar{k}}(\hat{\lambda}_f' + \hat{k}')
\end{aligned} \tag{8'}$$

$$\bar{\phi^T}\hat{\phi^T} = \bar{P_2}\bar{Y_2}(\hat{P_2} + \hat{Y_2}) - \bar{w_d}\bar{l_{d2}}(\hat{w_d} + \hat{l_{d2}}) - \bar{w_n}\bar{l_{n2}}(\hat{w_n} + \hat{l_{n2}}) - \bar{P_1}\bar{Y_1}(\hat{P_1} + \hat{Y_1}) + \bar{w_d}\bar{l_{d1}}(\hat{w_d} + \hat{l_{d1}}) \tag{9'}$$

$$\bar{L_d}\hat{L_d} = \frac{\bar{\phi^T}}{\bar{\phi} - \underline{\phi}}(\bar{l_{d2}} - \bar{l_{d1}})\hat{\phi^T} + \frac{\bar{\phi^T} - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{l_{d2}}\hat{l_{d2}} + \frac{\bar{\phi} - \bar{\phi^T}}{\bar{\phi} - \underline{\phi}}\bar{l_{d1}}\hat{l_{d1}} \tag{10'}$$

$$\bar{L_n}\hat{L_n} = \frac{\bar{\phi^T}}{\bar{\phi} - \underline{\phi}}\bar{l_{n2}}\hat{\phi^T} + \frac{\bar{\phi^T} - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{l_{n2}}\hat{l_{n2}} \tag{11'}$$

$$\hat{Y} = \bar{Y}^{-1/\lambda_f}\left(\frac{\bar{\phi}^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{Y_2}^{1/\lambda_f}\hat{Y_2} + \frac{\bar{\phi} - \bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\bar{Y_1}^{1/\lambda_f}\hat{Y_1} + \frac{\bar{\phi}^T}{\bar{\phi} - \underline{\phi}}(\bar{Y}_2^{1/\lambda_f} - \bar{Y}_1^{1/\lambda_f})\hat{\phi^T}\right)...$$

$$... + \left(\lambda_f \ln\left(\frac{\bar{\phi}^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{Y_2}^{1/\lambda_f} + \frac{\bar{\phi} - \bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\bar{Y_1}^{1/\lambda_f}\right) - \frac{\frac{\bar{\phi}^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{Y_2}^{1/\lambda_f}\ln(Y_2) + \frac{\bar{\phi} - \bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\bar{Y_1}^{1/\lambda_f}\ln(Y_1)}{\frac{\bar{\phi}^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\bar{Y_2}^{1/\lambda_f} + \frac{\bar{\phi} - \bar{\phi}^T}{\bar{\phi} - \underline{\phi}}\bar{Y_1}^{1/\lambda_f}}\right)\hat{\lambda_f} \tag{12'}$$

$$\bar{z}\bar{Y}(\hat{z} + \hat{Y}) = \bar{C}\hat{C} + \frac{\bar{\phi}^{T2}}{\bar{\phi} - \underline{\phi}}\hat{\phi}^T + \bar{k}\hat{k}' - (1-\delta)\bar{k}\hat{k} \tag{13'}$$

$$\hat{Y_1} = (1-\alpha)\hat{l_{d1}} + \alpha\hat{k} \tag{14'}$$

$$\bar{Y_2}\hat{Y_2} = (1-\alpha)\bar{k}^{\alpha}(\bar{l_{d2}}^{1-\alpha}\hat{l_{d2}} + \bar{l_{n2}}^{1-\alpha}\hat{l_{n2}}) + \alpha(\bar{l_{d2}}^{1-\alpha} + \bar{l_{n2}}^{1-\alpha})\bar{k}^{\alpha}\hat{k} \tag{15'}$$

$$\hat{q} = -\hat{C} \tag{16'}$$

Details for linearizing equation 8:

$$q = \beta\eta\frac{\partial}{\partial k'}E_{\phi}V(k', \phi') \tag{Start}$$

$$q = \beta\eta\frac{\partial}{\partial k'}E_{\phi}(q'(\max(V_1(k', \phi'), V_2(k', \phi'))) + \max_{k'}(q'(k'' - (1-\delta)k') + E_{\phi''}V(k''))) \tag{Subbing out V}$$

$$q = \beta\eta\frac{\partial}{\partial k'}\left(q'\left(\frac{\bar{\phi} - \phi^{T\prime}}{\bar{\phi} - \underline{\phi}}(P_1'Y_1' - w_d'l_{d1}') + \frac{\phi^{T\prime} - \underline{\phi}}{\bar{\phi} - \underline{\phi}}(P_2'Y_2' - w_d'l_{d2}' - w_n'l_{n2}' - (\phi^{T\prime} + \underline{\phi})/2)\right)...\right.$$

$$\left.... + \max_{k'}(q'(k'' - (1-\delta)k') + E_{\phi''}V(k''))\right) \tag{Filling out the EV and V1 and V2}$$

$$q = \beta\eta q'\left[\frac{-1}{\bar{\phi} - \underline{\phi}}(P_1'Y_1' - w_d'l_{d1}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\bar{\phi} - \phi^{T\prime}}{\bar{\phi} - \underline{\phi}}\left[\frac{\partial P_1'}{\partial k'}Y_1' + P_1'\frac{\partial Y_1'}{\partial k'}\right]...\right.$$

$$... + \frac{1}{\bar{\phi} - \underline{\phi}}(P_2'Y_2' - w_d'l_{d2}' - w_n'l_{n2}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\phi^{T\prime} - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\left[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}\right]...$$

$$\left.... - \frac{\phi^{T\prime}}{(\bar{\phi} - \underline{\phi})}\frac{\partial\phi^{T\prime}}{\partial k'} + (1-\delta)\right] \tag{Filling out the derivative}$$

$$q = \beta\eta q'\left[\frac{\underline{\phi}}{2(\bar{\phi} - \underline{\phi})}\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\bar{\phi} - \phi^{T\prime}}{\bar{\phi} - \underline{\phi}}\left[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}\right]...\right.$$

$$\left.... + \frac{\phi^{T\prime} - \underline{\phi}}{\bar{\phi} - \underline{\phi}}\left[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}\right] + (1-\delta)\right] \tag{Combining terms}$$

$$\frac{\partial P_i'}{\partial k'} = \left(\frac{1}{\lambda_f} - 1\right)\frac{P_i'}{Y_i'}\frac{\partial Y_i'}{\partial k'} \tag{Derivative of price}$$

$$\frac{\partial Y_i'}{\partial k'} = \alpha\frac{Y_i}{k} \tag{Derivative of Output}$$

$$q = \beta\eta q'\left[\frac{\alpha(\bar{\phi} - \phi^{T\prime})P_1'Y_1'}{\lambda_f(\bar{\phi} - \underline{\phi})k'} + \frac{\alpha(\phi^{T\prime} - \underline{\phi})P_2'Y_2'}{\lambda_f(\bar{\phi} - \underline{\phi})k'} + (1-\delta)\right] \tag{Subbing in Derivatives}$$

The last equation is what is linearized in 8'.

9.1 One Shift Version

$$\gamma L^{1/\theta_d} = \frac{w}{C} \tag{1}$$

$$z = P \tag{2}$$

$$L = (\frac{\lambda_f w Y^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{3}$$

$$q = \beta\eta \frac{\partial}{\partial k'} E_\phi V(k', \phi') \tag{4}$$

$$zY = C + k' - (1-\delta)k \tag{5}$$

$$Y = l^{1-\alpha} k^\alpha \tag{6}$$

$$q = \frac{1}{C} \tag{7}$$

Linearized:

$$\hat{\gamma} + \frac{1}{\theta_d}\hat{L} = \hat{w} - \hat{C} \tag{1'}$$

$$\hat{z} = \hat{P} \tag{2'}$$

$$\begin{aligned}\hat{L} =& \frac{\lambda_f}{1-\alpha-\lambda_f}(\hat{w} + \frac{1-\lambda_f}{\lambda_f}\hat{Y} - \hat{z}_t - \frac{\alpha}{\lambda_f}\hat{k}) + \frac{1-\alpha}{(1-\alpha-\lambda_f)^2}[\frac{1-\alpha-\lambda_f}{1-\alpha}... \\ & ... - (\frac{1-\alpha-\lambda_f}{\lambda_f(1-\alpha)} + \frac{1-\lambda_f}{\lambda_f})\ln\bar{Y} + (\frac{\alpha(1-\alpha-\lambda_f)}{2\lambda_f(1-\alpha)} + \frac{\alpha}{\lambda_f})\ln\bar{k}... \\ & ... - \ln\lambda_f - \ln\bar{w} + \ln(1-\alpha) + \ln\bar{z}]\hat{\lambda_f}\end{aligned} \tag{3'}$$

$$\hat{q} = \hat{\eta} + E[\hat{q}' + \beta\frac{\alpha}{\lambda_f}(\bar{P}\bar{l}^{1-\alpha}\bar{k}^{\alpha-1}(\hat{P}' + (1-\alpha)\hat{l}' + (\alpha-1)\hat{k}' - \hat{\lambda}'_f))] \tag{4'}$$

$$\bar{z}\bar{Y}(\hat{z} + \hat{Y}) = \bar{C}\hat{C} + \bar{k}\hat{k}' - (1-\delta)\bar{k}\hat{k} \tag{5'}$$

$$\hat{Y} = (1-\alpha)\hat{l} + \alpha\hat{k} \tag{6'}$$

$$\hat{q} = -\hat{C} \tag{7'}$$

9.2 Variation - fixed cost is labor:

$$\gamma L_d^{1/\theta_d} = \frac{w_d}{C} \tag{1}$$

$$\psi\gamma L_n^{1/\theta_n} = \frac{w_n}{C} \tag{2}$$

$$z(\frac{Y_1}{Y})^{1/\lambda_f - 1} = P_1 \tag{3}$$

$$z(\frac{Y_2}{Y})^{1/\lambda_f - 1} = P_2 \tag{4}$$

$$l_{d2} = (\frac{w_d}{w_n})^{-1/\alpha} l_{n2} \tag{5}$$

$$l_{n2} = (\frac{\lambda_f w_n (Y/((\frac{w_d}{w_n})^{1-1/\alpha} + 1))^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{6}$$

$$l_{d1} = (\frac{\lambda_f w_{dt} Y^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{7}$$

$$q = \beta\eta \frac{\partial}{\partial k'} E_\phi V(k', \phi') \tag{8}$$

$$\phi^T = (P_2 Y_2 - w_d l_{d2} - w_n l_{n2} - P_1 Y_1 + w_d l_{d1})/w_n \tag{9}$$

$$L_d = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{d2} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} l_{d1} \tag{10}$$

$$L_n = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} (l_{n2} + \frac{\phi^T + \underline{\phi}}{2}) \tag{11}$$

$$Y = (\frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} Y_2^{1/\lambda_f} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} Y_1^{1/\lambda_f})^{\lambda_f} \tag{12}$$

$$zY = C + k' - (1-\delta) * k \tag{13}$$

$$Y_1 = l_{d1}^{1-\alpha} k^\alpha \tag{14}$$

$$Y_2 = l_{d2}^{1-\alpha} k^\alpha + l_{n2}^{1-\alpha} k^\alpha \tag{15}$$

$$q = \frac{1}{C} \tag{16}$$

Linearizing 8 in the modified version:

$$
\begin{aligned}
q =& \beta\eta\frac{\partial}{\partial k'}E_\phi V(k',\phi') && \text{(Start)}\\
q =& \beta\eta\frac{\partial}{\partial k'}E_\phi(q'(\max(V_1(k',\phi'),V_2(k',\phi'))) + \max_{k'}(q'(k''-(1-\delta)k') + E_{\phi''}V(k''))) && \text{(Subbing out V)}\\
q =& \beta\eta\frac{\partial}{\partial k'}(q'(\frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline\phi}(P_1'Y_1'-w_d'l_{d1}') + \frac{\phi^{T\prime}-\underline\phi}{\bar\phi-\underline\phi}(P_2'Y_2'-w_d'l_{d2}'-w_n'l_{n2}'-w_n'\frac{\phi^{T\prime}+\underline\phi}{2}))...\\
&...+\max_{k'}(q'(k''-(1-\delta)k')+E_{\phi''}V(k''))) && \text{(Filling out the EV and V1 and V2)}\\
q =& \beta\eta q'[\frac{-1}{\bar\phi-\underline\phi}(P_1'Y_1'-w_d'l_{d1}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline\phi}[\frac{\partial P_1'}{\partial k'}Y_1' + P_1'\frac{\partial Y_1'}{\partial k'}]...\\
&...+\frac{1}{\bar\phi-\underline\phi}(P_2'Y_2'-w_d'l_{d2}'-w_n'l_{n2}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\phi^{T\prime}-\underline\phi}{\bar\phi-\underline\phi}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}]...\\
&...-\frac{\phi^{T\prime}}{\bar\phi-\underline\phi}w_n'\frac{\partial\phi^{T\prime}}{\partial k'} + (1-\delta)] && \text{(Filling out the derivative)}\\
q =& \beta\eta q'[\frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline\phi}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}]...\\
&...+\frac{\phi^{T\prime}-\underline\phi}{\bar\phi-\underline\phi}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}] + (1-\delta)] && \text{(Combining terms)}\\
\frac{\partial P_i'}{\partial k'} =& (\frac{1}{\lambda_f}-1)\frac{P_i'}{Y_i'}\frac{\partial Y_i'}{\partial k'} && \text{(Derivative of price)}\\
\frac{\partial Y_i'}{\partial k'} =& \alpha\frac{Y_i}{k} && \text{(Derivative of Output)}\\
q =& \beta\eta q'[\frac{\alpha(\bar\phi-\phi^{T\prime})P_1'Y_1'}{\lambda_f(\bar\phi-\underline\phi)k'} + \frac{\alpha(\phi^{T\prime}-\underline\phi)P_2'Y_2'}{\lambda_f(\bar\phi-\underline\phi)k'} + (1-\delta)] && \text{(Subbing in derivatives)}
\end{aligned}
\tag{15}
$$

9.3 Variation - fixed cost is higher depreciation:

$$\gamma L_d = \frac{w_d}{C} \tag{1}$$

$$\psi\gamma L_n = \frac{w_n}{C} \tag{2}$$

$$z(\frac{Y_1}{Y})^{1/\lambda_f - 1} = P_1 \tag{3}$$

$$z(\frac{Y_2}{Y})^{1/\lambda_f - 1} = P_2 \tag{4}$$

$$l_{d2} = (\frac{w_d}{w_n})^{-1/\alpha} l_{n2} \tag{5}$$

$$l_{n2} = (\frac{\lambda_f w_n (Y/((\frac{w_d}{w_n})^{1-1/\alpha} + 1))^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{6}$$

$$l_{d1} = (\frac{\lambda_f w_{dt} Y^{1/\lambda_f - 1}}{(1-\alpha) z k^{\alpha/\lambda_f}})^{\lambda_f/(1-\alpha-\lambda_f)} \tag{7}$$

$$q = \beta\eta \frac{\partial}{\partial k'} E_\phi V(k', \phi') \tag{8}$$

$$\phi^T = (P_2 Y_2 - w_d l_{d2} - w_n l_{n2} - P_1 Y_1 + w_d l_{d1})/k \tag{9}$$

$$L_d = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{d2} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} l_{d1} \tag{10}$$

$$L_n = \frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} l_{n2} \tag{11}$$

$$Y = (\frac{\phi^T - \underline{\phi}}{\bar{\phi} - \underline{\phi}} Y_2^{1/\lambda_f} + \frac{\bar{\phi} - \phi^T}{\bar{\phi} - \underline{\phi}} Y_1^{1/\lambda_f})^{\lambda_f} \tag{12}$$

$$zY = C + \frac{(\phi^{T2} - \underline{\phi}^2)}{2(\bar{\phi} - \underline{\phi})} k + k' - (1-\delta)k \tag{13}$$

$$Y_1 = l_{d1}^{1-\alpha} k^\alpha \tag{14}$$

$$Y_2 = l_{d2}^{1-\alpha} k^\alpha + l_{n2}^{1-\alpha} k^\alpha \tag{15}$$

$$q = \frac{1}{C} \tag{16}$$

Linearizing 8 in the modified version:

$$
\begin{aligned}
q =& \beta\eta\frac{\partial}{\partial k'}E_\phi V(k',\phi') && \text{(Start)}\\
q =& \beta\eta\frac{\partial}{\partial k'}E_\phi(q'(\max(V_1(k',\phi'),V_2(k',\phi'))) + \max_{k'}(q'(k''-(1-\delta)k') + E_{\phi''}V(k''))) && \text{(Subbing out V)}\\
q =& \beta\eta\frac{\partial}{\partial k'}(q'(\frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline{\phi}}(P_1'Y_1'-w_d'l_{d1}') + \frac{\phi^{T\prime}-\underline{\phi}}{\bar\phi-\underline{\phi}}(P_2'Y_2'-w_d'l_{d2}'-w_n'l_{n2}'-k'(\phi^{T\prime}+\underline{\phi})/2))...\\
& ... + \max_{k'}(q'(k''-(1-\delta)k') + E_{\phi''}V(k''))) && \text{(Filling out the EV and V1 and V2)}\\
q =& \beta\eta q'[\frac{-1}{\bar\phi-\underline{\phi}}(P_1'Y_1'-w_d'l_{d1}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline{\phi}}[\frac{\partial P_1'}{\partial k'}Y_1' + P_1'\frac{\partial Y_1'}{\partial k'}]...\\
& ... + \frac{1}{\bar\phi-\underline{\phi}}(P_2'Y_2'-w_d'l_{d2}'-w_n'l_{n2}')\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\phi^{T\prime}-\underline{\phi}}{\bar\phi-\underline{\phi}}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}]...\\
& ... - \frac{\phi^{T\prime}}{\bar\phi-\underline{\phi}}k'\frac{\partial\phi^{T\prime}}{\partial k'} + \frac{\phi^{T2\prime}-\underline{\phi}^2}{2(\bar\phi-\underline{\phi})} + (1-\delta)] && \text{(Filling out the derivative)}\\
q =& \beta\eta q'[\frac{\bar\phi-\phi^{T\prime}}{\bar\phi-\underline{\phi}}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}]...\\
& ... + \frac{\phi^{T\prime}-\underline{\phi}}{\bar\phi-\underline{\phi}}[\frac{\partial P_2'}{\partial k'}Y_2' + P_2'\frac{\partial Y_2'}{\partial k'}] + \frac{\phi^{T2\prime}-\underline{\phi}^2}{2(\bar\phi-\underline{\phi})} + (1-\delta)] && \text{(Combining terms)}\\
\frac{\partial P_i'}{\partial k'} =& (\frac{1}{\lambda_f}-1)\frac{P_i'}{Y_i'}\frac{\partial Y_i'}{\partial k'} && \text{(Derivative of price)}\\
\frac{\partial Y_i'}{\partial k'} =& \alpha\frac{Y_i}{k} && \text{(Derivative of Output)}\\
q =& \beta\eta q'[\frac{\alpha(\bar\phi-\phi^{T\prime})P_1'Y_1'}{\lambda_f(\bar\phi-\underline{\phi})k'} + \frac{\alpha(\phi^{T\prime}-\underline{\phi})P_2'Y_2'}{\lambda_f(\bar\phi-\underline{\phi})k'} + \frac{\phi^{T2\prime}-\underline{\phi}^2}{2(\bar\phi-\underline{\phi})} + (1-\delta)] && \text{(Subbing in derivatives)}
\end{aligned}
\tag{16}
$$

References

Altig, David, Lawrence J. Christiano, Martin Eichenbum, and Jesper Linde (2005) “Firm-Specific Capital, Nominal Rigidities, and the Business Cycle”, NBER Working Paper 11034

Basu, Susanto. 1996. “Procyclical Productivity: Increasing Returns or Cyclical Utilization?”, *The Quarterly Journal of Economics*, Vol. 111 (3), August, pp 719-751

Basu, Susanto and Miles S. Kimball. 1997. “Cyclical Productivity with Unobserved Input Variation”, NBER Working Paper 5915

Betancourt, Roger R. and Christopher K. Clague. 1981. *Capital Utilization.* Cambridge University Press

Bresnahan, Timothy F. and Valerie A. Ramey. 1993. “Segment Shifts and Capacity Utilization in the U.S. Automobile Industry”, *The American Economic Review*, Vol. 83 No. 2, Papers and Proceedings of the Hundred and Fifth Annual Meeting of the American Economic Association, (May, 1993), pp. 213-218

Bresnahan, Timothy F. and Valerie A. Ramey. 1994. “Output Fluctuations at the Plant Level”, *Quarterly Journal of Economics*, Vol. 109 No. 3, (Aug. 1994), pp. 593-624

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 1995. “Capital Utilization and Returns to Scale”, *NBER Macroeconomics Annual 1995, Volume 10*, pp. 67-124.

Chari, VV., Lawrence J. Christiano, and Patrick J. Kehoe. 1991. “Optimal Fiscal and Monetary Policy: Some Recent Results”, *Journal of Money, Credit, and Banking*, Vol. 23, No. 3, Part 2: Price Stability (Aug., 1991), pp. 519-539.

Christiano, Lawrence, Martin Eichenbaum, and Charles L. Evans (2005) “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *The Journal of Political Economy* 113(1)

Fiorito, Riccardo, and Giulio Zanella. 2008. "Labor Supply Elasticities: Can Micro Be Misleading for Macro?", *Government of the Italian Republic (Italy) - Ministry of Economy and Finance Department of the Treasury Working Papers Collection*

Golisov, Mikhail, and Robert E. Lucas, Jr. 2007. "Menu Costs and Phillips Curves", *Journal of Political Economy*, Vol. 115 No. 2, pp. 171-199

Hornstein, Andreas. 2002. "Towards a Theory of Capacity Utilization: Shiftwork and the Workweek of Capital", *Federal Reserve Bank of Richmond Economic Quarterly Volume 88/2 Spring 2002*, 65-86

Kostiuk, P. F. 1990. "Compensating Differentials for Shift Work", *Journal of Political Economy*, Vol. 98, No. 5, (October), pp. 1054-1075

Kydland, Finn E. and Edward C. Prescott. 1991. "Hours and Employment Variation in Business Cycle Theory," *Economic Theory*, Vol. 1, No. 1, (March), pp. 63-81

Lucas, Robert E., Jr. "Capacity, Overtime, and Empirical Production Functions," *The American Economic Review*, Vol. 60 No. 2, Papers and Proceedings, (May, 1970), pp. 23-27

Mattey, Joe, and Steve Strongin. 1997. "Factor Utilization and Margins for Adjusting Output: Evidence from Manufacturing Plants." Federal Reserve Bank of San Francisco *Economic Review 2*: 317.

Mayshar, Joram and Yorm Halevy. 1997. "Shiftwork", *Journal of Labor Economics*, Vol 15 No.1 Part 2: Esssays in Honor of Yoram Ben-Porath (Jan., 1997), ppp. S198-S222

Mayshar, Joram and Gary Solon. 1993. "Shift Work and the Business Cycle", *The American Economic Review*, Vol. 83, No. 2, Papers and Proceedings, (May, 1993), pp. 224-228

Ramey, Valerie A. and Daniel J. Vine. 2007. "Oil Shocks, Segment Shifts, and Capacity Utilization in the U.S. Automobile Industry: What has changes in 30 Years?", Working Paper.

Shapiro, Matthew D. 1993. "Cyclical Productivity and the Workweek of Capital," *The American Economic Review*, Vol. 83 No. 2, Papers and Proceedings of the Hundred and Fifth Annual Meeting of the American Economic Associations, (May, 1993), pp. 229-233

Shapiro, Matthew D. 1996a. "Capacity Utilization and the Marginal Premium for Work at Night." Unpublished paper. University of Michigan.

Shapiro, Matthew D., Carol Corrado, and Peter K. Clark. 1996b. "Macroeconomic Implications of Variation in the Workweek of Capital", *Brookings Papers on Economic Activity*, Vol. 1996, No. 2, pp 79-133

Smith, Kenneth R. 1970. "Risk and the Optimal Utilization of Capital", *The Review of Economic Studies* 37(2): 253-259